A Mathematical Model of the Airport Checkpoint

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The aim of this project is to model passenger throughput at airport checkpoints as a function of methods of liquid and gel (LAG) screening.

A variety of suitable liquid and gel scanners have been tested by the authorities and approved equipment has been categorised as Types A, B, C or D equipment. In Type C scanners, LAGs will be divested by passengers and placed on a tray prior to screening. There is no need to divest LAGs in a Type D scanner because they are screened *in situ*. This model examines passenger throughput as a function of both types of scanner.

The Mathematical Model

The model uses 21 separate variables to describe the effects on passenger throughput using. These variables are listed in Table 1. The functions are solved using a computer program and the data can be plotted as 2D and/or 3D graphs. It is not possible to run all of the variables simultaneously and present data that can be easily understood and so the data has been presented in the form of 3D surfaces where some variables are treated as constants while others range through a set of values.

To evaluate and illustrate the mathematical model, values have been assigned to the variables. These values are thought to be reasonable estimates but have not been measured at an airport. Some variables have been assigned a single value while others have been assigned a range of values. Some variables are treated as a constant in some evaluations and explored further as a range in other evaluations. The values and ranges were kept constant throughout this paper so that the different processes could be compared. It is recommended that measurements are collected at an airport checkpoint and are used to refine the findings.

Passenger throughput (pax) can be described as $Pax = \frac{Bags}{Pax_{bags}}$

All variables are defined in Table 1

The average number of items scanned each hour is impacted by the supplier listed scanner screening speed ($Scan_{rate}$), the passenger arrival rate (Pax_{arr}) and the passenger preparation time (Pax_{prep}) where

$$Pax_{prep} = Shed_t + Bin_t$$

Low passenger arrival rate

When $Pax_{arr} \times Pax_{bags} < Scan_{rate}$ (which is usually the case) then

$Bags = Pax_{arr} \times Pax_{bags}$ and in times of low arrival at a single checkpoint where

$Pax_{arr} \times (Shed_t+Bin_t)$ is < 1 hour then $Pax_{lo}=Pax_{arr}$ and queues should not be generated.

In a type D scanner there is no need to divest LAGs and so it can be assumed that passenger preparation ($Shed_t+Bin_t$) takes less time than in a Type C scanner. However, if an alarm is generated by a bag containing a liquid or gel then it is very probable that the regulator will require the bag to be re-screened once the LAG has been removed for further screening. Therefore Pax_{lo} in a Type D checkpoint system will be:

$Pax_{lo} = Pax_{arr}(1-zy)$ when $PaxD_{arr} \times (Shed_t+Bin_t)$ is < 1 hour*.

Where z is the bag reject rate (either the false positive rate of the scanner or the random selection percentage – whichever is the great number) and y is the percentage of bags that were found to contain LAGs and were therefore rescreened after the LAGs were removed for further screening. In a Type C scanner the equivalent equation is.

$$PaxC_{lo} = \frac{Pax_{arr} x Pax_{bags}}{(Pax_{bags} + Bin_{add})}$$ when $PaxC_{arr} \times (Shed_t + Bin_t + LAG_t)$ is < 1 hour*

Where LAG_t is the additional time required for the passenger to divest LAGs and place them into a bin and Bin_{add} has been added to take account of any additional bins which may be required.

Using the above equations and the following variable values a calculation was carried out. Assuming a passenger arrival rate from 1 pax/hr to the cut off values defined above (see *) and with total passenger preparation time ($Shed_t+Bin_t$) as 30s (for the removal of shoes (not slip on) and a coat or jacket, the removal of a laptop from a bag and the placement of a handbag and a larger bag on the conveyer). The average number of bags/items per passenger was defined as four (1 bin for the laptop, 1 bin for shoes and coat, 1 bin for handbag and any sundry items, and a free standing bag). The reject bag rate (z) was set at 0.1 and the percentage of alarm bags that need to be re-screened (y) at 33% of the alarm rate. The additional time, LAG_t, was been considered as two values: 2s and10s.

Maximum passenger throughput in a Type D system was found to be 116 pax/hr. Passenger throughput is eroded at a low rate because a proportion of bags are re-screened and this acts to reduce passenger throughput so that at a passenger arrival rate of 120 pax/hr the actual passenger throughput is 116 pax/hr. The same values were used in the Type C system computation but with the additional term Bin_{add} to take account if additional bins are required to hold the LAGs. Bin_{add} was varied between an overall bin increase of 0.4-0.9 bins per person. With a 2s LAG_t, passenger throughput is reduced and does not rise above 102 pax/hr when an additional bin time of 0.4 is used. If Bin_{add} is increased to 0.9 then passenger throughput drops to 91 pax/hr. If LAG_t is raised to 10s then the maximum passenger throughput at a Bin_{add} value of 0.4 is 82 pax/hr dropping to 74 pax/hr when Bin_{add} rises to 0.9.

The effect of these differences are that queues will start to form earlier in a Type C system because of the need to divest LAGs and the additional bins that might be required. Also, the low passenger arrival conditions finish at about 85 in a Type C system and at about 116 in a Type D system.

Higher passenger arrival rate

In this scenario it is assumed that $Pax_{arr} \times (Shed_t+Bin_t)$ is > 1 hour

but that the system is able to deal with the alarm resolution process without additional backlog. When the passenger rate and passenger preparation times are at this level, a queue will form and passenger throughput becomes defined by the time it takes to shed items and bin items i.e. $Pax_{hi} = 1/ Pax_{prep} = 1/(Shed_t+Bin_t)$. In the case of Type D scanners, throughput is additionally slowed by any rescreening of items after LAGs have been found in a bag selected for further screening producing:

$$PaxD_{hi} = \frac{(1-zy)}{(Shed_t + Bin_t)}$$

Calculations were carried out for a D type scanner with the reject bag rate set at 10%, the re-screening percentage ranging between 10-70% of the alarm bags and the combined $Shed_t$ and Bin_t time ranging between 15-45s. Note. The function shows that if the values used in the last section are used again (i.e. a re-screening rate of 33% and a combined $Shed_t$ and Bin_t time of 30s) then passenger throughput is computed as 116 pax/hr – the same throughput as computed at the top of the low passenger arrival range.

If the percentage of re-screened bags is only 10% of the reject bag rate, then with a passenger divest time held constant at 30s the passenger throughput only increases to 118 pax/hr. If the percentage of re-screened bags is increased to 70% then passenger throughput decreases to 112 pax/hr. These differences are fairly marginal. The form of the function means that changes in divest time makes a greater difference. If the percentage of re-screened bags is held constant at 33% then passenger throughput with an average divest time of 15s is 232 pax/hr. If the divest time is increased to 45s then passenger throughput drops by 67% to 77 pax/hr. So the function is very sensitive to average passenger $Shed_t$ and Bin_t times.

In the case of a type C scanner, the passenger preparation time factor is exacerbated by the additional time required to divest and bin LAGs but, on the other hand, rescreening is not required. Another important factor is the growth of bins if additional bins are required to hold the LAGs.

$$PaxC_{hi} = \frac{1}{(shed_t + bin_t + LAG_t)} * \frac{Pax_{bags}}{(Pax_{bags} + Bin_{add})}$$

So taking the same range for passenger preparation time as previously and adding a LAG_t component ranging between 2-10s, a Bin_{add} value of 0.7 and keeping the value of Pax_{bags} as 4 we see that with a passenger preparation time of 30s and a LAG_t time of 2s, passenger throughput becomes 96 pax/hr – slightly more than the top of the low passenger arrival range. If LAG_t is increased to 10s then passenger throughput drops to 78 pax/hr. If passenger preparation time is reduced to 15s then passenger throughput becomes 180 with a LAGt of 2s and 125 if LAGt is increased to 10s. If passenger preparation time is kept constant at 30s and LAG_t is kept constant at 5s but Bin_{add} is varied from 0.4-0.9 then passenger throughput changes from 94 pax/hr at a Bin_{add} value of 0.4 and drops to 84 at a Bin_{add} value of 0.9.

This shows that any increase in the divest time i.e. by adding 2-10s for divesting liquids, has an impact on passenger throughput. Likewise, if the liquids need to be put into a separate bins or cause other items to be put in extra bins, then this too will slow passenger throughput.

Note. A simplifying term can be considered at this point because no matter what factors are in control of passenger arrival and preparations the average passenger throughput at the scanner approximates to the average passenger throughput through the AMD. This assumes that there are no hold ups in the AMD process itself and does not apply to a portal where passenger throughput is typically lower than through an AMD. In this case:

$$Pax_{hi}=AMD_{thru}$$

showing that a slowdown in AMD throughput is a strong indicator of problems in passenger preparation for the scanner.

Alarm resolution

Another potential contributor to queuing at the checkpoint is the time required for alarm resolution. In type D scanners, the false alarm or random selection rate has two components: alarm bags that contain LAGs and alarm bags that do not contain LAGs. The hourly alarm resolution time for a Type D scanner ($Alarm_d$) can therefore be written as:

$$Alarm_D = Pax.Pax_{bags}.z((1-y)Search + y.LAG_D) \quad \text{(hrs)}$$

where Search is the time required to take a case from the line, take to a search table, call the passenger and carry out a manual search; y is the fraction of alarm bags that contain a LAG and LAG_d is the time taken to resolve a bag containing one or more LAGs and where

$$LAG_d = Prep_d + LAG1(LAG2 + LAG3) \quad \text{(hrs)}$$

where $Prep_d$ is the time required to take a case from the line to a search table, call the passenger, open the bag and finally return it to the scanner for rescreening. LAG1 is the number of LAGS in a bag, LAG2 is the time to find and remove each LAG and LAG3 is the time to screen each LAG using another technology and return to bag. If there are only the resources (staff and/or space) at a given checkpoint, for the resolution of one bag at a time, then the time spent resolving alarms in an hour ($Alarm_d$) must be less than or equal to 1 hour in order to prevent a backlog building up. For this to happen and taking account of a variable number of alarm resolution stations:

$$1hr \times Bagspace \text{ needs to be} \geq Pax.Paxbags.z((1-y)Search + y.LAG_d) \quad \text{(hrs)}$$

where Bagspace is the number of alarm resolution stations, so

$$LAG_d \text{ needs to be} < \frac{1}{y}\left[\frac{Bagspace}{Pax.Paxbags.z} - (1-y)Search\right] \quad \text{(hrs)}$$

for the system to remain in balance. The hourly alarm resolution time for a Type C scanner ($Alarm_c$) can be written as:

$$Alarm_c = Pax.Paxbags.z.Search + Pax.Bin_{add}.z.LAG_c \quad \text{(hrs)}$$

where for simplicity it has been assumed that all LAGs are placed in a separate bin and the false alarm or random selection rate is the same for bags and LAGs and where LAG_c = LAG1.LAG3. The time spent resolving alarms in an hour ($Alarm_c$) must be less than or equal to 1 hour in order to prevent a backlog building up. For this to happen and taking account of a variable number of alarm resolution stations:

$$1hr \times Bagspace \text{ needs to be} \geq Pax.Pax_{bags}.z.Search + Pax.Bin_{add}.z.LAG_c \quad \text{(hrs)}$$

So

$$LAG_c \text{ needs to be} < \left[\frac{Bagspace}{Pax.Paxbags} - z.Search\right].\frac{Pax_{bags}}{Bin_{add}.z} \quad \text{(hrs)}$$

for the system to remain in balance.

Alarm resolution in balance

When the alarm resolution time does not take longer than the bag screening time and the system remains in balance. Any queues are caused by a passenger arrival time that is too high to be absorbed by the system. Therefore passenger throughput remains as previously stated.

Alarm resolution forced into balance

Should

$$LAG_d > \frac{1}{y}\left[\frac{Bagspace}{Pax.Paxbags.z} - (1-y)Search\right]$$

in a Type D system or

$$LAG_c > \left[\frac{Bagspace}{Pax.Paxbags} - z.Search\right].\frac{Pax_{bags}}{Bin_{add}.z}$$

in a Type C system, the alarm bags will build up and a backlog will eventually clog the system. Clearly the more resources that are put aside for alarm resolution stations, the less likely this is to happen. If alarm resolution time does take too long then passenger throughput becomes controlled by the rate of alarm resolution. This is because the system can only clear the reduced number of bags that will produce the maximum reject bags that can be screened by the alarm resolution stations. Thus the system will force itself into balance again.

So we can compute the number of bags that can be alarm resolved in an hour i.e. what reduction in Bags pushes the system back into balance. This gives us a revised passenger throughput for a Type D scanner:

$$PaxD_{hi} = \frac{Bagspace}{zy.Paxbags\left[\left[\frac{1}{y}-1\right]Search+LAG_d\right]}$$

This is the second factor that can cause the queue to grow and also the form of the equation now shows that passenger shed and bin times become irrelevant because passenger throughput is entirely governed by the alarm resolution times.

Calculation for a Type D system (See fig.1a) shows that passenger throughput is 200 pax/hr when the average LAGs per person are just 2 and the secondary screening time is just 20s. If the average number of LAGs is increased to 3 and the alarm resolution time to 30s then the passenger throughput falls to 143 pax/hr. This is a higher passenger throughput than computed using passenger divest times indicating that using these values the scanner and alarm resolution methods are able to deal with the passenger flow without creating additional queuing. If the average number of LAGs is 3 but the alarm resolution time increases to 45s then passenger throughput drops to 115 pax/hr - about the same as the top of the low passenger arrival rate and the mid passenger throughput rates. Thus, again, no additional queuing will occur but if the average number of LAGs is retained as 3 and secondary screening time increases to 1 min then passenger throughput drops to 97 pax/hr and the result will be a slowdown at the scanner caused by backlog and leading to additional queuing.

In fact there are many variable combinations where passenger throughput drops below that allowed by passenger loading times. These are shown in figure 1 where the flat part of the graph shows the area of variable combination where additional queues will be generated by backlog from the scanner. Figures 4(b)-(d) show the effect of adding in additional alarm resolution stations. These reduce the potential for queue production due to backlog and allow the system respond to higher passenger flow rates across a range of variable values.

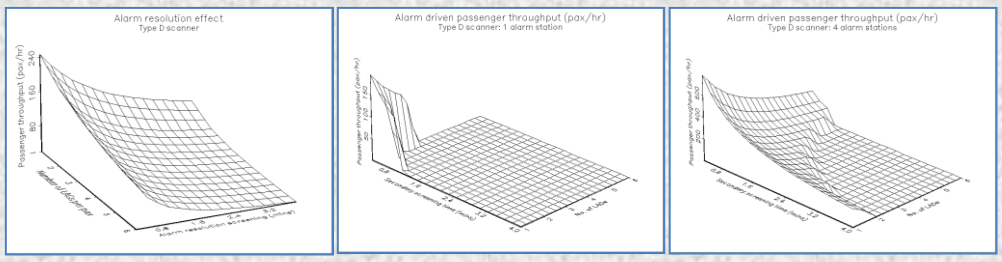

Figure 1. Type D scanner results showing (a) how passenger throughput varies with alarm resolution time and number of LAGs per pax and (b) - (c) variable combinations where a backlog is generated in a Type D system with 1 and 4 alarm resolution stations respectively. Note. The flat area shows the variable space where backlog will occur.

In the case of a Type C scanner the revised passenger throughput is:

$$PaxC_{hi} = \frac{Bagspace}{z((Paxbags.Search)+(Bin_{add}.LAG_c))}$$

Calculation shows that passenger throughput starts as 243 with an average LAGs per person of 2 and an secondary screening time of 20s (see fig.2a). This is approximately 20% more than the throughput in a Type D scanner using the same values. If the average number of LAGs is increased to 3 and the secondary screening time to 30s then the passenger throughput falls to 197 pax/hr – approximately 38% more than a Type D scanner. If alarm resolution increases to 1 min then passenger throughput drops to 146 pax/hr - an increase of 51% on a Type D scanner. Thus a Type C scanner consistently has a higher throughput than a Type D scanner and the comparative gain in throughput increases with the time required for secondary screening - although the throughput itself is decreased.

The best passenger loading times were computed as 90-100 pax/ per hour and it can be seen that most variable combinations lead to a system that can cope with these loading rates. Figure 2 shows passenger throughput as a function of the variable space for a Type C scanner with just one alarm resolution station and it can be clearly seen that less combinations of LAG number and secondary screening time will lead to backlog and additional queuing than in a Type D scanner system.

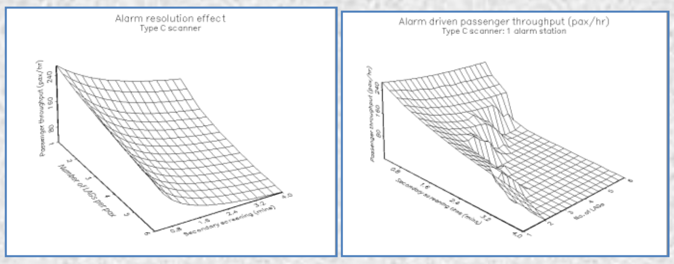

Figure 2. Showing (a) how passenger throughput varies with alarm resolution time and number of LAGs per pax in a Type C system and (b) with one alarm resolution station where the flat area of the graph represents variable combinations where a backlog is generated.

However, this is partially misleading because the loading rate in a Type C system is lower than in a Type D system i.e. if we were to use the Type C loading figures in the calculation producing fig.4 then the flat area would be somewhat reduced. Also because passenger loading in a Type C scanner system peaks at lower passenger throughputs then, no matter how fast the alarm resolution is, this fact will dominate the end result because no scanner system can work faster than the loading pace.

AMD and Portal effects

In a smoothly running checkpoint, the passengers will pass through the AMD at about the same pace as the scanner loading rate and the AMD does not unduly affect passenger screening rate. The introduction of portals may change this balance. If passenger throughput through the portal (Pax_{port}) is less than the bag loading rate at the scanner, then bags will start to accumulate on the conveyer and system throughput will become controlled by portal throughput i.e.

$$Pax = Pax_{port}$$

this would start to take effect if portal throughput dropped below about 116 in a Type D scanner or around 85 in a Type C scanner when using the variable values used in these calculations. This is the third source of queues.

Summary and conclusions

- There are three main contributing factors to queues and backlog at the checkpoint:
    - Passenger divest times
    - Alarm resolution effect
    - Portal/AMD throughput
- There are differences in Type C and Type D scanner systems that impact on average passenger preparation time leading to different cut off points for the 'no-queue', low passenger arrival condition. In a Type D system (using values given in Table 1) passengers are unlikely to be affected by queues until passenger arrival rates become greater than about 116 pax/hr while this point is reached at the much earlier point of ~85 pax/hr when using a Type C system.
- As passenger arrival rates increase, queues form and passenger throughput becomes dependent on passenger divest and bin preparation times. Passenger throughput remains at a similar level and the differences in passenger throughput through C and D systems are maintained. These differences are mainly due to the extra time required to divest LAGs in a Type C system and the requirement for additional bins. Re-screening bags, as might occur in a Type D system, has a relatively minor effect on passenger throughput.
- Passenger arrival rate and scanner loading times determines the maximum throughput of both systems. This is because no system can work faster than its loading pace. However, alarm resolution has the ability to reduce passenger throughput.
- Alarm resolution in either Type C or D systems is able to slow passenger throughput. Alarm resolution takes longer in a Type D system because the LAGs have to be divested before they can undergo secondary screening – thus a Type D system has more potential to slow passenger throughput than a Type C scanner. This can be offset by opening more alarm resolution stations.
- A Type C system has a lower ceiling on its throughput but this ceiling is less likely to be compromised by backlog during the alarm resolution process.
- Passenger throughput can be maximised by either using a Type D system in combination with several alarm resolution stations or by finding new ways to reduce passenger preparation time in order to increase the loading rate in either system.

The introduction of portals to the checkpoint may play a future role in passenger throughput. If the time taken to collect an image and inspect it is longer than passenger divest and bin preparation time, then passenger throughput will be impacted.